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Mathematical model of friction wear process on the example of polymer composites on the basis of phenol-formaldehyde resins

Summary — Mathematical expression allowing calculation of temperature (*T*) in the friction zone was found by use of similarity theory and dimensions analysis. This temperature is a criterion of abrasion estimation [friction (abrasive) wear]. The model has been elaborated and on its basis one can calculate *T* value of any element being a part of friction pair. The condition of usefulness of a model is being familiar with mechanical and thermal characteristics of a polymer composite as well as its work conditions.
Key words: friction wear, mathematical model, polymer composite, friction zone temperature.

It is known the reliability and durability of the friction pairs define exploitation efficiency of any industrial equipment. Malfunction of friction pair leads to premature wear of an equipment, to technology strictness transgression, to exceeding energy expenses, to unreasonable increase in the friction, and to violation of the safety and hygiene of work conditions. So, the friction wear is one of the most important features of the trybologic object. Namely, it defines mainly the efficiency, reliability and safety of friction pairs.

The polymer composites are used more frequently for the friction pairs of modern machines. The reasons are: increase in reliability of the machines, improvement in their working features. Refusing the noble metals alloys, being in short supply, let decrease machines' prices. Perspective using of polymer materials for the friction pairs is very closely connected to the progress in our knowledge about their physico-mechanical properties and elaboration of their efficiencies criteria. One of them is the friction wear [1].

An ability to predict wear of the friction pairs under given exploiting conditions provides an optimum choice of materials and specific design of the friction pair. To calculate the wear value some mathematical models are used. This way it is possible to decrease the time and material expenses for the friction pairs' design [2].

KINETIC MODEL OF FRICTION PROCESS

We chose a kinetic model to describe the friction wear of a polymer composite material. This model is based on the thermo-fluctuation concept for solid body strength developed by S. N. Jurkov et al. [3]. According to the concept, the disintegration of material looks like a time developing thermo-activation process (τ) that is described by such fundamental equation:

$$\tau = \tau_0 \cdot e^{\frac{U_0 - \gamma \cdot \sigma}{RT}} \qquad (1)$$

where: τ — *time of disintegration starting from applying the load (life-time, longevity),* τ_0 — *constant value (the oscillation period of atoms),* U_0 — *activation energy needed to destroy the inner interconnections of a material,* σ — *destroying stress value,* RT — *heat movement's energy,* γ — *some constant value defined by the material's structure.*

From the point of view of kinetic theory the surface layer is considered as a body that consists of a great number of structure elements — kinetics units. They could be the material's atom and molecules submitted heat oscillations.

Equation (1) assumes that material parameters (U_0, γ, τ_0) are constant. However, there is temperature (melting point) over which the constancy disappears. Then the unlimited increase in temperature inevitably will result in loss of the form of a body or its integrity. Hence, there is some boundary temperature, characteristic for the given material, above which the equation (1) loses the sense.

Really, the experiences have shown, that instead of the equation (1) it is necessary to use the following equation [4]:

$$\tau = \tau_m \cdot \exp\left[\frac{U_0 - \gamma \cdot \sigma}{RT}\left(1 - \frac{T}{T_m}\right)\right] \qquad (2)$$

where: τ — *minimum durability of a material, characterized as much as possible with temperature* T_m*, at which at any* *loading or without in the material softens owing to intensive break of intermolecular connections or collapses as a result of break of internuclear connections [5].*

Kinetic model use enables to calculate the wear of polymers. It must be taken into account that according to the kinetic model the wearing process is connected with destruction and separation of tiny particles. And so, it is not possible to fix the start time at the moment when load was applied and finish one with separation of certain particle. Similar role as durability plays resistance to wear *i.e.* time of branch of certain weight of a material. Therefore in the equation (2) for wear instead of τ it is necessary to put $1/I$ where I describes the value of wear. But it is more convenient to use simply I not its reciprocal. Then equation (2) assumes the following form:

$$I = I_m \cdot \exp\left[-\frac{U_0 - \mu \cdot \gamma \cdot \sigma}{RT}\left(1 - \frac{T}{T_m}\right)\right] \quad (3)$$

where: I_m — *wear constant i.e. single-action wear that is going on during every separate mechanical act of interaction,* μ — *friction factor.*

Let us note τ_m from equation (2) is a minimal value of the life-time when elementary disintegration is going on during one act of mechanical interaction, that is under every atoms' oscillation. In parallel with τ_m the constant I_m in equation (3) shows a critical value when disintegration is going with very large speed, and when temperature or/and stress are so high that timing or an activation-kinetic description are not applicable for the disintegration process.

There is parameter I in the equation (3) — the friction temperature — which is rather sensitive to the working conditions and can vary in a wide range [6]. The physico-mathematical modeling along with the similarity and dimensions analyzing theory were used to develop an analytical relations defining the temperature within a friction zone for the polymer composite material based on phenol-formaldehyde resin [7].

Let us consider changing of the polymer composite's temperature in the case of friction process. This is a function of space and time coordinates:

$$T = f(x, y, z, \tau) \quad (4)$$

where: x, y, z — *space coordinates of a body,* τ — *time coordinate.*

Relations of these variables are described by the Fourier differential equation:

$$\frac{\partial T}{\partial \tau} = b\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (5)$$

where: $b = \lambda/c$ — *thermo-conductivity coefficient (coefficient of heat transfer inside the body) of the polymer material (in* m^2/s*),* λ — *thermo-conductivity coefficient weighted-average for all materials placed along the heat current's path toward the friction surface,* c — *thermal capacity coefficient average-weighted for all materials participating in the heat transmission process.*

To solve the equation (5) the first moment's temperature distribution (as a start condition) as well as geometrical shape of the body and how the friction pairs surface interacts with outer environment (boundary condition) must be known. Following types of the boundary conditions could be applied to the friction pairs:

— Temperature distribution along the body surface — defined as a function of time, $T_\pi(\tau) = f(\tau)$. In a special case (of the static temperature field) $T_\pi(\tau) = T_\pi = const$, so, the surface temperature is kept the same during the whole friction process.

— Heat flow through the body surface — defined as a function of time, $q_\pi(\tau) = f(\tau)$. The simplest case when this flow does not change, $q_\pi(\tau) = q_\pi = const$.

— Relation of the convection heat exchange between the body surface and outer environment is defined so to keep the constant heat flow value:

$$q_\pi = \alpha(T_c - T_\pi) = -\lambda\frac{dT}{dx} \quad (6)$$

where: α — *thermo-transmission coefficient (coefficient of heat transfer to the outer environment, in* $W/m^2 \cdot deg$*),* T_c — *ambient temperature,* T_π — *temperature of a body.*

CORRELATION BETWEEN EXPERIMENTAL DATA AND BASIC KINETIC EQUATION

General solution of (5) is an infinite set of all possible particular solutions. The exact solution of this equation could be found only if some simplifications are applied, but it is rather difficult to predict them. Of course, looking at the main values and investigating their relations in direct experiments could help. But the experiment can not distinguish, in general, particular features and common ones (applicable to many phenomenon). Similarity theory provides the common solution of this task [8]. It establishes relations between experimental data directly connected with the basic equations, and so clarifies their common features. Practically, in mathematical analysis of heat processes the complex dimensionless variables could be easily used instead of primary ones.

Criteria-functions

We are operating with complexes that consist of known data or contain the variables that we are looking for. Common view of the initial equation's solution could be presented in the form of criteria as a dependence of criteria-functions on some criteria-arguments.

The general factors that influence the friction could be described by following dependences, where: M — mass, L — length, τ — time and T — temperature - is a basic unit measurement.

— Outer mechanical parameters: the relative velocity of friction of surfaces $V = L \cdot \tau^{-1}$ (in m/s), the load $P = M \cdot L^{-1} \cdot \tau^{-2}$ (in MPa).

— Thermal characteristic: friction temperature depending on the quantity of heat evolved during friction $q = M \cdot L^2 \cdot \tau^{-3}$, heat-transfer (from the surface) coefficient value $\alpha = \tau^{-3} \cdot T^{-1}$, heat-engineering properties of the materials: heat-conduction $\lambda = M \cdot L \cdot \tau^{-3} \cdot T^{-1}$, heat storing capacity $c = L^2 \cdot \tau^{-2} \cdot T^{-1}$.

— Physico-mechanical properties of the conjugate pairs impact strength according $a = L^2 \cdot M \cdot \tau^{-2}$, (in kJ/m^2), ultimate bending strength $\sigma_b = M \cdot L^{-1} \cdot T^{-2}$ (in MPa), ultimate compression strength $\sigma_c = M \cdot L^{-1} \cdot \tau^{-2}$ (in MPa); density $\rho = M \cdot L^{-3}$ (in kg/m^3).

— Macro- and microgeometry of the friction face: d — basic sample size L (in cm), h — roughness value of the fricting surfaces L (in cm). Some characteristic and shape of the conjugate pairs contacting area and also their surroundings.

— Friction type: sliding friction, rolling friction and a combination of these types.

— Friction time and track.

The complex criteria can be received not only using the mathematical description of the phenomena and can based only on the analysis of dimensions of sizes essential to the given process.

If the geometrical parameters of friction pair, loading — speed mode of operations, thermo — physical and physico-mechanical characteristics of a material are given, it is possible to determine temperature in a zone of friction as a function:

$$T = f(P, V, \rho, \sigma_b, \sigma_c, a, m, d, \alpha, c, \lambda) \quad (7)$$

where: m — *modulus of surface of material considerated (its volume: surface ratio).*

Theory of dimensions and similarity theory

As plenty of friction pairs modern machines work at the limited lubrication, we shall consider a definition of temperature mode of operations of friction pairs working without greasing. The function (7) includes 12 variable sizes at 4 basic units of dimensions. In this case there should be 12 - 4 = 8 of the dimensionless relations between parameters of expression (7). It is possible to make from the above mentioned parameters in (7) a number of the dimensionless relations and to attribute them to the basic function, not applying the analysis of the theory of dimensions. To such criteria belong: $\left(\frac{\sigma_b}{\sigma_c}\right); \frac{d}{m}; Nu = \frac{\alpha \cdot d}{\lambda}$ — criterion of Nusselt. It contains size, determined in calculation of α — heat-transfer coefficients. The number Nu is the dimensionless form heat-transfer coefficient and characterizes intensity of process of heat exchange by contact between a body surface and environment.

Then the required dependence will accept if:

$$T = f\left(P, V, \rho, a, m, d, c, \alpha, \frac{\sigma_b}{\sigma_c}, \frac{d}{m}, \frac{\alpha \cdot d}{\lambda}\right) \quad (8)$$

Applying to other sizes the theory of dimensions, we shall receive 5 dimensionless relations. And, for function it is possible to accept any five sizes. As it is known [8], in case of unknown quantity (n) of equations and exceeding number (m), (n - m) independent variables exist. On the basic of it we set the parameters T, P, V, c, d equal serially l, equating parameters of other sizes to zero.

On the basic of expression (8), we receive the following kind of the equations:

$$T = \rho^{x_1} \cdot a^{y_1} \cdot m^{z_1} \cdot \alpha^{n_1} \quad (9)$$

$$P = \rho^{x_2} \cdot a^{y_2} \cdot m^{z_2} \cdot \alpha^{n_2} \quad (10)$$

$$V = \rho^{x_3} \cdot a^{y_3} \cdot m^{z_3} \cdot \alpha^{n_3} \quad (11)$$

$$c = \rho^{x_4} \cdot a^{y_4} \cdot m^{z_4} \cdot \alpha^{n_4} \quad (12)$$

$$d = \rho^{x_5} \cdot a^{y_5} \cdot m^{z_5} \cdot \alpha^{n_5} \quad (13)$$

Let's express these equations through symbols of dimensions:

$$T = (M \cdot L^{-3})^{x_1} \cdot (M \cdot L^2 \cdot \tau^{-2})^{y_1} \cdot L^{z_1} \cdot (M \cdot \tau^{-3} \cdot T^{-1})^{n_1} \quad (14)$$

$$P = M \cdot L^{-1} \cdot \tau^{-2} = (M \cdot L^{-3})^{x_2} \cdot (M \cdot L^2 \cdot \tau^{-2})^{y_2} \cdot L^{z_2} \cdot (M \cdot \tau^{-3} \cdot T^{-1})^{n_2} \quad (15)$$

$$V = L \cdot \tau^{-1} = (M \cdot L^{-3})^{x_3} \cdot (M \cdot L^2 \cdot \tau^{-2})^{y_2} \cdot L^{z_3} \cdot (M \cdot \tau^{-3} \cdot T^{-1})^{n_3} \quad (16)$$

$$c = L^2 \cdot \tau^{-2} \cdot T^{-1} = (M \cdot L^{-3})^{x_4} \cdot (M \cdot L^2 \cdot \tau^{-2})^{y_4} \cdot L^{z_4} \cdot (M \cdot \tau^{-3} \cdot T^{-1})^{n_4} \quad (17)$$

$$d = L = (M \cdot L^{-3})^{x_5} \cdot (M \cdot L^2 \cdot \tau^{-2})^{y_5} \cdot L^{z_5} \cdot (M \cdot \tau^{-3} \cdot T^{-1})^{n_5} \quad (18)$$

Equating parameters at appropriate units of dimensions, we shall receive 5 groups of the equations with four unknown quantities:

$$\left.\begin{array}{l} x_1 + y_1 + n_1 = 0 \\ -3x_1 + 2y_1 + z_1 = 0 \\ -2y_1 - 3n_1 = 0 \\ -n_1 = 1 \end{array}\right\} \quad (19)$$

$$\left.\begin{array}{l} x_2 + y_2 + n_2 = 1 \\ -3x_2 + 2y_2 + z_2 = -1 \\ -2y_2 - 3n_2 = -2 \\ -n_2 = 0 \end{array}\right\} \quad (20)$$

$$\left.\begin{array}{l} x_3 + y_3 + n_3 = 0 \\ -3x_3 + 2y_3 + z_3 = 1 \\ -2y_3 - 3n_3 = -1 \\ -n_3 = -1 \end{array}\right\} \quad (21)$$

$$\left.\begin{array}{l} x_4 + y_4 + n_4 = 0 \\ -3x_4 + 2y_4 + z_4 = 2 \\ -2y_4 - 3n_4 = -2 \\ -n = -1 \end{array}\right\} \quad (22)$$

$$\left.\begin{array}{l} x_5 + y_5 + n_5 = 0 \\ -3x_5 + 2y_5 + z_5 = 1 \\ -2y_5 - 3n_5 = 0 \\ -n_5 = 0 \end{array}\right\} \quad (23)$$

Having decided of the systems of the equations (19)—(23) (the theory of determinants can give the answer at once), we shall receive the following meanings of unknown quantifies:

$x_1 = -\frac{1}{2}$; $x_2 = 0$; $x_3 = -\frac{1}{2}$; $x_4 = -\frac{1}{2}$; $x_5 = 0$;

$y_1 = \frac{3}{2}$; $y_2 = 1$; $y_3 = \frac{1}{2}$; $y_4 = -\frac{1}{2}$; $y_5 = 0$;

$z_1 = -\frac{9}{2}$; $z_2 = -3$; $z_3 = -\frac{3}{2}$; $z_4 = \frac{3}{2}$; $z_5 = 1$;

$n_1 = -1$; $n_2 = 0$; $n_3 = 0$; $n_4 = 1$; $n_5 = 0$.

Having addressed them to expressions (9)—(13) and substituting the found meanings of parameters, we shall receive that the dimensionless correlations will have the quite certain kind, namely:

$$T = \rho^{-0.5} \cdot a^{1.5} \cdot m^{-4.5} \cdot \alpha^{-1} \quad (24)$$

$$P = a \cdot m^{-3} \quad (25)$$

$$V = \rho^{-0.5} \cdot a^{0.5} \cdot m^{-1.5} \quad (26)$$

$$c = \rho^{-0.5} \cdot a^{0.5} \cdot m^{1.5} \cdot \alpha \quad (27)$$

$$d = m \quad (28)$$

Dependence of temperature on factors analyzed

The general dependence of temperature in a zone of friction can be submitted as:

$$T = \frac{a^{1.5}}{\rho^{0.5} \cdot m^{4.5} \cdot \alpha} \cdot f\left(\frac{P \cdot m^3}{a}; \frac{V \cdot \rho^{0.5} \cdot m^{1.5}}{a^{0.5}}; \frac{m^{1.5} \cdot \alpha}{c \cdot \rho^{0.5}}; \frac{\sigma_b}{\sigma_c}; \frac{d}{m}; \frac{\alpha \cdot d}{\lambda} \right) \quad (29)$$

Due to the theory of similarity and analysis of dimensions we have had an opportunity to establish a degree of dependence of temperature on the factors analyzed. The size of function (7) should be defined by experience, and for each friction pair will be different. Expression (29) is an obvious kind of experimentally investigated dependence of temperature on loading and on speed of sliding of polymeric composites on a basis the phenol-formaldehyde resins. To answer the purpose the temperature was measured by microthermocouples, which have been closed up in the samples 0.3—0.4 mm far from a surface of friction. The results of measurements were registered by a potentiometer.

$$T = A \cdot \frac{a^{1.5}}{\rho^{0.5} \cdot m^{4.5} \cdot \alpha} \cdot \left(\frac{P \cdot m^3}{a} \right)^{k_1} \cdot \left(\frac{V \cdot \rho^{0.5} \cdot m^{1.5}}{a^{0.5}} \right)^{k_2} \cdot \left(\frac{m^{1.5} \cdot \alpha}{c \cdot \rho^{0.5}} \right)^{k_3} \cdot \left(\frac{\sigma_b}{\sigma_c} \right)^{k_4} \cdot \left(\frac{d}{m} \right)^{k_5} \cdot \left(\frac{\alpha \cdot d}{\lambda} \right)^{k_6} \quad (30)$$

Having expressed the equation (29) in the form more convenient for use, we receive (equation (30)).

Taking into account, that some parameters of equation (30) can be the same for model and real object, the values of exponents in the appropriate dimensionless correlations will be equal to zero and the whole appropriate element of equation (30) will become one. For example, when σ_b and σ_c values are the same for model and real object the exponent $k_4 = 0$ and element $(\sigma_b / \sigma_c)^{k_4} = 1$.

If in the model both material and nature are the same and ratio of geometrical parameters are identical, criterion of the dependence (30) can be transformed, and will look like:

$$T = A \cdot \frac{a^{1.5}}{\rho^{0.5} \cdot m^{4.5} \cdot \alpha} \cdot \left(\frac{P \cdot m^3}{a} \right)^{k_1} \cdot \left(\frac{V \cdot \rho^{0.5} \cdot m^{1.5}}{a^{0.5}} \right)^{k_2} \quad (31)$$

Having transformed experimental data it is possible to determinate numerical meanings of factor A and exponents k_1 and k_2 for (31) with the help of computation.

CONCLUSIONS

So, there was developed a mathematical model based on the similarity theory and kinetic model of the friction wear. The model let calculate the wear value for friction pairs polymer made of composites when their working parameters and basic data of the composites are known. We believe that model use let save both time and materials in comparison with a real friction wear experiments in real conditions.

LITERATURE

1. Kogaev V. P., Drozdov Yu. N.: "Prochnost' i iznosostoikost detalei mashin", Vyshaya Shola, Moscov 1991.
2. "Osnovy tribologii" (Ed. Chichinadze A. V.), Nauka i t'ekhnika, Moscov 1995.
3. Berkovich I. I., Gromakovskii D. G.: "Tribologiya. Fizicheskie osnovy, mekhanika i tekhnicheskie prilozheniya: Uchebnik dlya vuzov", Samarskii Gosudarstvennyii Tekhnicheskii Uniwersitet, Samara 2000.
4. Ratner S. B.: *Plast. Massy* 1990, No 6, 35.
5. Ratner S. B., Yartsev V. P.: "Fizicheskaya mekhanika plastmass. Kak prognoziruyut rabotosposobnost' ?", Khimiya, Moscov 1992.
6. Tretyakov A. O., Burmistr V., Ovtsharov V. I., Bashtanik R. I.: "Verrechnung der Arbeitsfähigkeit von Compositen auf Basis von Phenol-Formaldehyd-Bindemittel in den Reibungsknoten", Technometr 2001, Chemnitz (Germany), Conf. Mat., 197.
7. Tretyakov A. O., Bashtannik R. I., Burmistr' M. V.: *Vopr. Khim. Khim. Tekhnol.* 2001, No 1, 123.
8. Sedov L. I.: "Metody podobiya i razmernosti v mekhanike", Nauka, Moscov 1987.

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